GCE Examinations

Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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1. Given that $y = \mathrm{e}^{\arctan x}$,

(a) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **(4 marks)**

The curve $y = \mathrm{e}^{\arctan x}$ has a point of inflexion.

(b) Find the coordinates of this point of inflexion. **(3 marks)**

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2. *(a)* Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2 - 1}}.$$ **(3 marks)**

(b) Find

$$\int \operatorname{arcosh} x \ \mathrm{d}x.$$ **(4 marks)**

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3. Find

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin 2x} \ \mathrm{d}x.$$ **(8 marks)**

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4. *(a)* Find

$$\int \frac{1}{\sqrt{4x^2-4x+10}}\,\mathrm{d}x\,.$$ **(6 marks)**

(b) Hence evaluate

$$\int_{\frac{1}{2}}^{2} \frac{1}{\sqrt{4x^2-4x+10}}\,\mathrm{d}x\,,$$

giving your answer in terms of natural logarithms. **(3 marks)**

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5. *(a)* On the same axes sketch the curves with equations $y = 2 - \tanh x$ and $y = 3\,\mathrm{sech}\,x$, giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes.

(5 marks)

(b) Solve the equation

$$2 - \tanh x = 3\,\mathrm{sech}\,x,$$

giving your answers to 2 decimal places. **(7 marks)**

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6. $$I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,\mathrm{d}x, \qquad n \geq 0.$$

(a) Show that

$$I_n = \frac{n-1}{n} I_{n-2}, \qquad n \geq 2.$$ **(7 marks)**

The curve C is defined by $y = \sin^2 x,\ 0 \leq x \leq \pi$.

The area bounded by C and the positive x-axis is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated giving your answer in terms of π. **(7 marks)**

Turn over

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7.

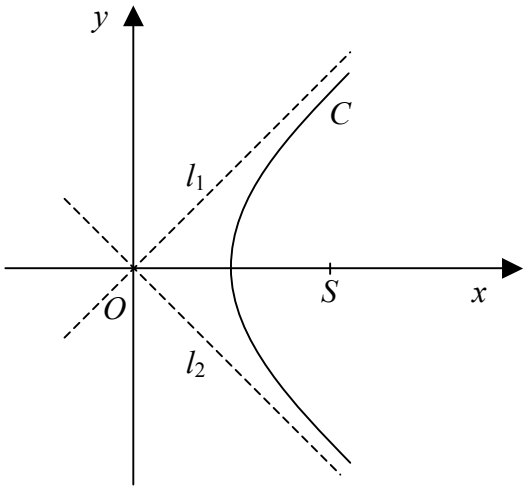


Fig. 1

Figure 1 shows the curve C which is part of the hyperbola with parametric equations

$$x = a\cosh t\,, \qquad y = 2a\sinh t\,,$$

where a is a positive constant and $x \geq a$. The lines l_1 and l_2 are asymptotes to C.

(a) Show that the radius of curvature of C at its vertex is $4a$. **(6 marks)**

(b) Show that an equation of the tangent to C at the point $P(a\cosh p,\ 2a\sinh p)$ is

$$2x\cosh p - y\sinh p = 2a\,.$$ **(4 marks)**

The tangent to the curve C at P meets the asymptote l_1 at Q.

Given that QS is parallel to the y-axis, where S is the focus,

(c) show that $p = \frac{1}{2}\ln 5$. **(8 marks)**

END

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